A fractional cointegration approach to testing the Ohlson accounting based valuation model

Shih-Cheng Lee, Chien-Ting Lin & Min-Teh Yu
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A fractional cointegration approach to testing the Ohlson accounting based valuation model

Shih-Cheng Lee · Chien-Ting Lin · Min-Teh Yu

Abstract We examine the long-run relationship between market value, book value, and residual income in the Ohlson (Contemp Acc Res 11(2):661–687, 1995) model. In particular, we test if market value is cointegrated with book value and residual income in light of their non-stationary behaviors. We find that cointegration applies to only 51% of the sample firms, casting doubt that book value and residual income alone are adequate in tracking variations in market value, yet we find that market value is fractional cointegrated with book value and residual income for 89% of the sample firms. This implies that the long-run relationship follows a slow but mean-reverting process. Our results therefore support the Ohlson model.

Keywords Accounting based valuation · The Ohlson model · Value relevance · Residual income · Book value

JEL Classification G17 · M40

1 Introduction

Ever since the seminal work of Ohlson (1995) on residual income valuation (RIV) model, several studies have examined the empirical validity of the model. Although the Ohlson model or its variants are theoretically derived from the discounted cash flow model or dividend discount model, Penman (2001) argues that the valuation of a firm’s equity in practice involves forecasting over finite and truncated horizons. As a result, the choice of
cash flow or accrual accounting matters as it affects the estimation of payoffs over the finite horizon and hence the valuation of a firm.

There are two general findings in the literature. First, the Ohlson model explains stock prices better than discounted cash flow models (e.g. dividend discount model) over truncated, finite horizons (see Bernard (1995), Callen and Morel (2005), and Jiang and Lee (2005)). Penman and Sougiannis (1998) demonstrate that the former yields lower valuation errors, because accrual accounting provides a correction to discounted cash flow valuation. The correction includes the recognition of non-cash value changes and matching the cost of investment against inflows of investment in time through depreciation allocations. Such treatments facilitate valuing firms from the forecast of payoffs especially over short horizons.

Second, Frankel and Lee (1998) contend that the Ohlson model provides a more complete valuation approach than popular alternatives (e.g. the dividend discount model). Callen and Morel (2005) highlight the Ohlson model by pointing out that valuing non-dividend paying firms is made possible by replacing dividends with (abnormal) earnings. It therefore makes the accounting-based valuation approach appealing especially when the proportion of firms paying cash dividends in the US fell from 66.5% in 1978 to 20.8% in 1999. In sum, the RIV model has arguably become the workhorse for accounting-based valuation in recent years (Penman (2005)).

Qi et al. (2000) and Callen and Morel (2005) document that a firm’s market value and book value exhibit non-stationary (non-mean reverting) behavior. Their findings suggest that OLS regressions may yield spurious relations with large $R^2$ and significant $t$-statistics even when market value, book value, and residual income are independent (or not related). To mitigate the spurious results, Qi et al. (2000) examine the relationship using the cointegration method, but they find that market value is cointegrated with book value and residual income for only 25% of their sample firms. Their evidence therefore does not lend support to the Ohlson model as book value and residual income fail to adequately capture time variation in stock prices.¹

One possible reason for the low cointegration of market value with both book value and residual income may be related to the omission of “other information” in the Ohlson model. Ohlson (1995) suggests that the “other information” in the model should be thought of as summarizing value relevant events that have yet to have an impact on the financial statements (p. 668). Such information bears upon future (abnormal) earnings independent of current and past (abnormal) earnings. In other words, the “other information” captures all non-accounting information that will be eventually reflected in future abnormal earnings. It follows that book value and residual income are “slow” in tracking time varying and forward looking market value due to the omitted information. The opaque nature of non-accounting information may delay the response of book value and residual income to the movement of market value before it is fully revealed in the intrinsic value.

An obvious remedy for the Ohlson model is to specify what the “other information” is, but Ohlson (1995) leaves us little clue as to what it might contain. Consequently, there are several attempts to identify the “other information”. They include consensus analyst forecasts of next year’s earnings (Dechow et al. (1999)), order backlog (Myers (1999)), dividends (Hand and Landsman (1998)), accruals and cash flows (Barth et al. (1999)), research and development expenses (Callen and Morel (2005)), pension liabilities

¹ The results reported by Qi et al. (2000) are based at the 10% significant level. The percentage of firms for which market value is cointegrated with book value and residual income will be less than 25% at the 5% significant level.
A fractional cointegration approach

(Gopalakrishnan (1994)), derivative disclosures (Wang et al. (2005)), and audit and non-audit fees (Brown and Caylor (2006)).

While this line of approach potentially fills the missing link in the Ohlson model, the focus has inevitably turned into whether the “other information” has been correctly specified and how well it fits into the model. The essence and the spirit of the dynamic Ohlson process for relating market value to book value and residual income is largely ignored or forgotten. Equally important, given that several factors have been identified as the “other information” so as to modify future residual income, the aggregation of these other information poses a challenge in itself. Myers (1999) suggests that it is not possible to control for all the “other information”.

In the spirit of the Ohlson model and with the view that we cannot fully specify what the “other information” is, we examine whether a firm’s book value and residual income track its market value in a slow but cointegrated process. Specifically, we test if the adjustment of book value and residual income to the movement of market value is partially complete (due to the lag effect of the “other information”), differentiating from the standard cointegrated process implied by Qi et al. (2000). In other words, the co-movement with market value may require longer time than that implied by full cointegration as other information about future residual income working its way into the accounting system.

This depiction of the process for slow adjustment to equilibrium is consistent with Ohlson (1995), who suggests that the information dynamics between current accounting numbers and future residual income are part of the important chain of information that ultimately feeds into market value. Therefore, rather than using the full cointegration analysis applied by Qi et al. (2000), we estimate fractional cointegration that allows for partial co-movement between market value, book value, and residual income. This approach of examining the Ohlson model is also appealing, because it avoids specifying and aggregating the “other information”.

Our empirical investigation shows that although market value is not fully cointegrated with book value and residual income, it is, however, fractional cointegrated. This implies that the relationship specified in the Ohlson model follows a slow but mean-reverting (or stationary) process. Our results therefore support the Ohlson model in which a firm’s book value and residual income alone are adequate in explaining its market value over the long run.

The remainder of the paper is organized as follows. Section 2 briefly reviews the Ohlson model and its empirical tests. Section 3 discusses the fractional cointegration methodology. Section 4 describes the sample and defines the variables in our tests. We present the empirical results in Sect. 5 and conclude the paper in Sect. 6.

2 Prior tests of the Ohlson model

Based on the dividend discount model in which the market value of a firm’s equity is the present value of expected future dividends, Ohlson (1995) expresses the firm valuation in terms of the expected future residual income:

\[ P_t = BV_t + \sum_{s=1}^{\infty} R^{-s} E_t(RI_{t+s}) \]  

(1)

where \( P_t \) is the market value, or price, of the firm’s equity at time \( t \), \( BV_t \) is (net) book value at time \( t \), \( R \) is the cost of equity capital plus one, and \( E_t(RI_{t+s}) \) is the expected residual
income in period $t + \tau$. Equation (1) is often known as the residual income valuation (RIV) model since a firm’s future profitability as measured by the present value of anticipated residual income reconciles the difference between market and book values.

To complete the valuation analysis, Ohlson (1995) assumes an information dynamic linkage between current information and future residual income in an autoregressive process:

$RI_{t+1} = \omega RI_t + v_t + \varepsilon_{1t+1}$

$v_{t+1} = \gamma v_t + \varepsilon_{2t+1},$

where $v_t$ is information other than residual income, $\varepsilon_{1t}$ and $\varepsilon_{2t}$ are mean-zero disturbance terms, and $\omega$ and $\gamma$ are parameters that are non-negative and less than 1. The market value of a firm’s equity in Eq. (1) becomes:

$P_t = BV_t + \alpha_1 RI_t + \alpha_2 v_t,$

where $\alpha_1 = \omega / (R - \omega) \geq 0$ and $\alpha_2 = R / (R - \omega) (R - \gamma) \geq 0$. Equation (4) shows that the market value, $P_t$, depends on the book value adjusted for the current residual income, $RI_t$, and the other information, $v_t$, that modifies the prediction of future profitability.$^2$

Beaver (1999) argues that the Ohlson model has simplicity and generality properties, because it only requires book value and expected residual income to explain market value and can be applied in any accounting system in which a clean surplus relation holds. It is also important to note that the specifications from Eqs. (1) to (3) are in a time-series relationship. Therefore, testing the validity of the model should also be consistent with the time-series relation between market value, book value, and residual income. However, some earlier studies examine the relation between stock prices (or stock returns) and earnings (e.g. Frankel and Lee (1998), Penman and Sougiannis (1998), Dechow et al. (1999), and Tsay et al. (2008)), in cross-sections. As a result, conducting cross-sectional tests on the Ohlson model fails to take into account the time-varying changes in time-dependent explanatory variables.

For studies that test the time-series relationship of a firm’s market value on book value, earnings, and other value relevant variables, the non-stationarity of stock price and explanatory variables may lead to spurious relations that yield large $R^2$ and significant $t$-statistics even if they are not related (see Granger and Newbold (1974), Phillips (1987), Morel (2003), Wong and Chan (2004), and Liu et al. (2012)). Consistent with the non-stationarity properties of market value and book value, Qi et al. (2000) find that only 5.3 % of market values and 1.1 % of book values in their sample firms exhibit stationary behavior. Despite this important econometric issue, Myers (1999) and Ahmed et al. (2000) conduct their empirical tests using OLS regressions without first examining the non-stationary properties of their data.

Engle and Granger (1987) and Johansen (1988) show that OLS regressions with non-stationary time-series data yield consistent estimates when the relationship between dependent variable and independent variables are said to be cointegrated. Conceptually, cointegration implies that there exists a long-run relationship in a set of variables. In our context, it suggests that even when market value, book value, and residual income are individually non-stationary, market value can still be related to book value and residual income in a stable process over the long run. Statistically speaking, the error terms in a cointegration process are stationary even though the dependent variable and some of the independent variables are not. Based on the

$^2$ For the detailed derivation of the Ohlson Model, refer to Ohlson (1995).
cointegration framework, Qi et al. (2000) find that only 25% of market values in their sample firms are cointegrated with both book value and residual income. Their results therefore do not lend support for the Ohlson model.

### 3 Fractional cointegration

One common approach to solve the non-stationarity problem in a regression is by taking differences (mostly in the first difference) on the variables. However, this method tends to destroy valuable information that may exist in a long-term relationship between variables. Granger (1986) and Engle and Granger (1987) develop the standard cointegration method for which OLS estimates are consistent and establish the long-run equilibrium relationship between data series even when an individual series is non-stationary and has infinite variance. Specifically, each variable in the model is first tested for its non-stationarity and the order of integration. If all the variables are found to be integrated of the same order, assuming integrated of order 1 (I(1)), then these variables are said to be cointegrated if the residual series \( \hat{e}_t \) in the following equation is stationary or integrated or order 0 (I(0)). Algebraically:

\[
y_t = \beta_0 + \beta_1 z_t + e_t,
\]

where \( y_t \) and \( z_t \) are dependent and independent variables, respectively.

Fractional cointegration, the generalized version of cointegration, examines the long-run equilibrium relationship on a fractional domain. Given a shock to an equilibrium system, a return to the equilibrium may take place over a long horizon if the deviations from equilibrium follow a fractionally integrated process. The non-stationary integrated time series may therefore be fractionally cointegrated where deviations from equilibrium may not be integrated of order zero, but rather integrated in the order between zero and one.

Applying the fractional cointegration method to the Ohlson (1995) model, we first establish if market value, book value, and residual income are non-stationary and are integrated of order 1 (I(1)). We then regress market value on book value and residual income over the sample period. The unit root tests (non-stationarity tests) are then carried out on the error terms in the OLS regressions based on an autogressive fractional integrated moving average (ARFIMA) \((p, d, q)\) model where \(0 < d < 0.5\);

\[
\Phi(L)(1 - L)^d(y_t - \mu) = \Theta(L)u_t, \quad u_t \sim i.i.d.(0, \sigma_u^2).
\]

The term \( L \) is the backward-shift operator, \( \Phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p \), \((1 - L)^d\) is the fractional differencing operator, \( y_t \) is the fractional integrated process, \( \mu \) is a constant, \( \Theta(L) = 1 + \theta_1 L + \cdots + \theta_q L^q \), and \( u_t \) is an I(0) process. Since Diebold and Rudebusch (1991) report that the Augmented Dickey Fuller (ADF) unit root test tends to carry low power of rejecting non-stationarity, we use the Gaussian semiparametric test of Robinson (1995) for the non-stationarity test. “Appendix” provides a detailed discussion on the methodology.

### 4 Sample and variable measurement

Annual financial data are obtained from CRSP and the Compustat industrial file between 1973 and 2006. To be included in the sample, a firm must have a closing stock price at
fiscal year-end, a total number of shares outstanding, and net income over the sample period. Its book value of equity should be non-negative in any year. We further require that a firm has lagged book value in order to calculate residual income. We exclude banks, securities firms, and insurance firms due to their unusual debt level and regulatory constraints. The criteria yield a final sample of 314 firms over 34 years of the sample period.

All variables are reported on a per share basis. We define market value of common equity \( (MV_t) \) as the closing stock price at the end of year \( t \). Residual income, \( RI_t \), is calculated for each firm in each year as:

\[
RI_t = NI_t - r_tBV_{t-1},
\]

where \( NI_t \) is the net income before extraordinary items in year \( t \), \( r_t \) is the cost of equity capital at \( t \), and \( BV_t \) is the book value at \( t \). In estimating residual income, Dechow et al. (1999) use a constant discount rate or an industry risk premium similar to Fama and French (1997). Lee et al. (1999) argue that firm value estimates track stock prices more closely when using a time-varying component of the cost of capital. Therefore, we implement industry cost of capital from Fama and French (1997) and the cost of equity capital from the Fama and French three-factor model to estimate residual income.

We next run fractional cointegration tests on the following Ohlson models using two residual income estimates for robustness checks:

\[
P_t = a + \beta_1BV_t + \beta_2RI_{1t} + e_t, \tag{8}
\]

\[
P_t = a + \beta_1BV_t + \beta_2RI_{2t} + e_t, \tag{9}
\]

where \( P_t \) is the market value, \( BV_t \) is the book value, \( RI_{1t} \) is the residual income based on the industry risk premium of Fama and French (1997) to determine cost of equity capital, and \( RI_{2t} \) is the residual income based on the Fama and French three-factor model to determine the cost of equity capital. If either cost of equity capital is less than the 10-year treasury-bill rate, a proxy for the risk-free rate, then we use the risk-free rate for the cost of equity capital.

It is important to note that to derive the Ohlson model, one must assume the clean surplus relation (CSR) that equates the change in book value to earnings minus dividends. This assumption allows dividends to be replaced by book value and residual income in Eqs. (8) or (9).

In testing the validity of the model, we follow Dechow et al. (1999) and McCrae and Nilsson (2001) in their cross-sectional studies, and Callen and Morel (2001) and Ota (2002) in their time-series studies by excluding extraordinary items from net income as they cause an unstable estimation of the linear information model. Dechow et al. (1999) argue that although such treatment of net income violates the CSR assumption, extraordinary items are non-recurring in practice. Therefore, their inclusion is unlikely to enhance the prediction of abnormal earnings.

The CSR violation is in general often observed since the GAAP’s earnings construct allows value relevant accounting items to be charged directly to the book value without showing up in earnings. To check if the violation of the CSR may affect the outcome of our tests, we estimate the degree of dirty surplus accounting for firms in which their market values are cointegrated with book value and residual income and for firms in which their market values are not cointegrated. We find that firms with cointegration exhibit higher dirty surplus accounting than those without cointegration (results are not tabulated). This therefore suggests that the CSR violations have a minimum impact on testing the validity of the Ohlson model.
5 Empirical results

5.1 Descriptive statistics

Table 1 presents the descriptive statistics of the sample from 1973 to 2006. The average market value of the sample firms is $32.07 per share, or more than four times larger than the average book value of $7.64 per share. However, the difference between the median market value of $25.99 per share and book value of $5.03 per share is larger than that between the average market and book values. As expected, market value is more volatile (a standard deviation of 33.39) than book value (a standard deviation of 13.19) as the former regularly updates the market expectation of a firm’s value while the latter measures the historical value of equity, therefore suggesting that book value alone may not be sufficient to track the variation of market value over time.

Comparing the two estimates of residual incomes, $RI_1^t$, based on the industry risk premium of Fama and French (1997), the cost of equity capital is consistently higher than $RI_2^t$, based on the Fama and French 3-factor model across all quartiles. In particular, an average firm earns $0.96 per share using $RI_1^t$ and $0.70$ per share using $RI_2^t$. It implies that the cost of equity capital for $RI_1^t$ tends to be lower than that for $RI_2^t$ to generate higher residual income. As the estimate of residual income is sensitive to a particular measure of the cost of equity capital, we estimate the time-series relationship between firm value and residual income under both measures.

5.2 Non-stationarity test

Before we estimate if market value is cointegrated with book value and residual income in the Ohlson model, we test the non-stationarity of each of the variables at level and the stationarity at first difference. Table 2 presents the summary statistics of Augmented Dickey-Fuller (ADF) unit root tests without the time trend at level and at first difference. The null hypothesis is that the Ohlson variables contain a unit root and are therefore non-stationary. The alternative hypothesis indicates stationarity if the test statistic is less than the critical value of $-2.93$ at the 5% level.

Panel A of Table 2 shows that at level, the mean (median) statistic is $-2.54$ ($-2.57$) for market value and $-1.34$ ($-1.76$) for book value respectively. The results indicate that both variables for an average sample firm are non-stationary. As a proportion of sample firms,
the last column in Panel A reports that only 35 and 23 % of market value and book value in the sample firms are stationary, respectively. For the residual incomes, $RI_1^t$ and $RI_2^t$, the stationarity improves to 61 and 64 % of the sample firms, respectively. While residual incomes exhibit relative higher stationarity than market value and book value, the overall results are nevertheless consistent with Qi et al. (2000), who conclude that both market and book values are non-stationary.

After taking the first difference of market value, book value, and residual income, Panel B of Table 2 shows that all of the variables have become stationary. More specifically, 97 % of market values, 83 % of book values, and 99 and 97 % of residual incomes $RI_1^t$ and $RI_2^t$, respectively, of the sample firms are stationary. According to the preliminary results, market value, book value, and residual income for most of the firms can be characterized by a process of integrated order of 1 (I(1)).

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
<th>Percentage of stationarity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Level</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market value</td>
<td>-2.54</td>
<td>1.31</td>
<td>-3.26</td>
<td>-2.57</td>
<td>-1.93</td>
<td>35.35</td>
</tr>
<tr>
<td>Book value</td>
<td>-1.34</td>
<td>2.36</td>
<td>-2.87</td>
<td>-1.76</td>
<td>-0.43</td>
<td>23.25</td>
</tr>
<tr>
<td>Residual income 1 ($RI_1^t$)</td>
<td>-3.34</td>
<td>1.27</td>
<td>-4.21</td>
<td>-3.27</td>
<td>-2.41</td>
<td>60.51</td>
</tr>
<tr>
<td>Residual income 2 ($RI_2^t$)</td>
<td>-3.33</td>
<td>1.37</td>
<td>-4.20</td>
<td>-3.30</td>
<td>-2.61</td>
<td>64.01</td>
</tr>
<tr>
<td><strong>Panel B: First difference</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market value</td>
<td>-6.48</td>
<td>1.94</td>
<td>-7.55</td>
<td>-6.29</td>
<td>-5.49</td>
<td>97.45</td>
</tr>
<tr>
<td>Book value</td>
<td>-5.15</td>
<td>2.90</td>
<td>-6.90</td>
<td>-5.59</td>
<td>-3.97</td>
<td>82.80</td>
</tr>
<tr>
<td>Residual income 1 ($RI_1^t$)</td>
<td>-7.10</td>
<td>1.97</td>
<td>-8.20</td>
<td>-6.92</td>
<td>-5.74</td>
<td>98.73</td>
</tr>
<tr>
<td>Residual income 2 ($RI_2^t$)</td>
<td>-7.03</td>
<td>1.96</td>
<td>-8.06</td>
<td>-7.09</td>
<td>-5.92</td>
<td>97.13</td>
</tr>
</tbody>
</table>

This table presents the summary statistics of the ADF test for the Ohlson factors at the level and at the first difference. The critical value for the Augmented Dickey-Fuller unit root test without trend at the 5 % level is -2.93. The last column of each panel reports the percentage of firms that are stationary at the 5 % significant level.

5.3 Cointegration test of the Ohlson model

As a sequel to establishing that market value, book value, and residual income follow I(1) process, we examine whether and the extent to which they are cointegrated. Panels A and B of Table 3 present the cointegration and OLS results for the two estimates of residual income. The last column in both panels shows that only about half of the market values of the sample firms are cointegrated with their book values and residual incomes (53.5 % for $RI_1^t$ and 50.1 % for $RI_2^t$). The findings are consistent under both measures of residual incomes and are therefore insensitive to the estimates of the cost of equity capital.

It is important to note that while the cointegration results do not lend support for the Ohlson model, both book value and residual income in the OLS results (see column 1) are statistically significant in explaining market value. However, as discussed earlier, these results may not be reliable as they could yield a spurious relationship specified in the Ohlson model. In the following sub-section, we examine whether the Ohlson model can best be characterized by a fractional cointegration (slow but mean-reverting) process.
5.4 Fractional cointegration of the Ohlson model

A fractional cointegration implies that the parameter of integration $d$ is less than one (for full cointegration, $d = 1$). To estimate $d$ for the residuals of the Ohlson model, we follow the Gaussian Semi-Parametric method of Robinson (1995). Hurvich et al. (1998) suggest that the optimal bandwidth parameter to minimize the mean squared error is when the power of $l$ is 0.8. For robustness checks, we estimate the parameter of integration $d$ using a range of power of $l$ from 0.70 to 0.90 with an interval of 0.05.

Table 4 provides a brief interpretation of value $d$. For the Ohlson model to follow a slow but mean-reverting process, the range of $d$ should fall between 0 and less than 1. In particular, if $d$ is greater than 0 but equal or less than 0.5, then the process is said to be fractionally cointegrated (stationary). However, if it is greater than 0.5 but less than 1, then the process is mean reverting but not stationary.

Panels A and B of Table 5 report the results for fractional cointegration estimates over a range of $\mu$. We find that the estimates of $d$ are consistently greater than 0 but less than 1 across different powers of $\mu$ at the 1 and 5 % significant level, indicating that the process in which variations in market value are explained by book value and residual income is slow but mean-reverting. These results are robust to both measures of residual income. Furthermore, since $d$ is always less than 0.5, the fractional cointegrated process is stationary.

As the power of $\mu$ increases, the proportion of sample firms in which market value is fractional cointegrated with book value and residual income improves monotonically.

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Table 3  Regression and cointegration analysis of the Ohlson model

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
<th>Percentage of stationary</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: RI1</strong></td>
<td></td>
<td></td>
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<tr>
<td>Intercept</td>
<td>20.35***</td>
<td>15.43</td>
<td>9.47***</td>
<td>17.50***</td>
<td>27.33***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.98)</td>
<td>(2.86)</td>
<td>(4.68)</td>
<td>(6.45)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Book value</td>
<td>1.08***</td>
<td>1.24</td>
<td>0.41</td>
<td>0.91***</td>
<td>1.42***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.57)</td>
<td>(1.08)</td>
<td>(2.47)</td>
<td>(3.98)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residual income</td>
<td>2.58***</td>
<td>3.01</td>
<td>0.61</td>
<td>1.90*</td>
<td>3.63***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.01)</td>
<td>(0.70)</td>
<td>(1.79)</td>
<td>(3.08)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{ADF}$</td>
<td>−3.60</td>
<td>1.11</td>
<td>−4.26</td>
<td>−3.60</td>
<td>−2.99</td>
<td>53.50</td>
</tr>
<tr>
<td><strong>Panel B: RI2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>21.45***</td>
<td>15.61</td>
<td>10.85***</td>
<td>19.03***</td>
<td>28.57***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.34)</td>
<td>(3.17)</td>
<td>(5.00)</td>
<td>(6.91)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Book value</td>
<td>1.05***</td>
<td>1.26</td>
<td>0.36</td>
<td>0.92***</td>
<td>1.49***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.46)</td>
<td>(0.90)</td>
<td>(2.33)</td>
<td>(3.65)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residual income</td>
<td>1.98*</td>
<td>2.77</td>
<td>0.27</td>
<td>1.24</td>
<td>2.89***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.65)</td>
<td>(0.35)</td>
<td>(1.41)</td>
<td>(2.75)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{ADF}$</td>
<td>−3.59</td>
<td>1.09</td>
<td>−4.19</td>
<td>−3.54</td>
<td>−2.95</td>
<td>50.96</td>
</tr>
</tbody>
</table>

This table presents the regression and cointegration results of the Ohlson model, $MV_t = \alpha + \beta_1 BV_t + \beta_2 RI_t + \epsilon_t$, where $MV_t$ is the market value at time $t$, $BV_t$ is book value at time $t$, and $RI_t$ is the residual income at time $t$. $t$-statistics are reported in parentheses. $T_{ADF}$ is the test statistics of Augmented Dickey Fuller (ADF) test. The critical value for ADF unit root test without trend at the 5 % level is −3.52 (Mackinnon (2010)). $t$-statistics are reported in the parentheses. * and *** denote statistical significance at the 10 and 1 % level, respectively.

---

3 See “Appendix” for a discussion of the Gaussian semi-parametric method.
When $l = 0.90$, the proportion of samples firms that are fractional cointegrated reaches a maximum of 87% (in Panel A) or 89% (in Panel B) at the 5% significant level. In sum, our empirical investigations demonstrate that book value and residual income are adequate in capturing variations in market value. Our results therefore support the Ohlson model. However, the relationship specified can best be described in a fractional cointegration framework where tracking the movement of a firm’s market value by the corresponding movement in book value and residual income is a slow but mean-reverting process.

**6 Conclusion**

In this study we investigate the empirical validity of the Ohlson model in which market value is a function of book value and residual income. Under the non-stationary behavior of these variables, we test the relationship using a cointegration approach to address the

### Table 4 Interpretation of Value $d$ (differencing)

<table>
<thead>
<tr>
<th>Value $d$</th>
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</tr>
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<tbody>
<tr>
<td>&gt;1.0</td>
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This table presents the range of the order of integration, $d$, from 0 to greater than 1 and its relation to testing the Ohlson model.

### Table 5 Gaussian semiparametric estimates of the fractional differencing parameter $d$ for the Ohlson model (H0: $d = 0$ vs. H1: $d \neq 0$)

<table>
<thead>
<tr>
<th>Power order of $\mu$</th>
<th>$d$</th>
<th>$t$-stat</th>
<th>1 %</th>
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<tbody>
<tr>
<td><strong>Panel A: RI1</strong></td>
<td></td>
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<tr>
<td>0.70</td>
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The table presents the results of fractional cointegration of the Ohlson model over a range of order of $\mu$. Market value is said to be fractional cointegrated with book value and residual income when $d$ varies between 0 and 0.5.

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In sum, our empirical investigations demonstrate that book value and residual income are adequate in capturing variations in market value. Our results therefore support the Ohlson model. However, the relationship specified can best be described in a fractional cointegration framework where tracking the movement of a firm’s market value by the corresponding movement in book value and residual income is a slow but mean-reverting process.

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In this study we investigate the empirical validity of the Ohlson model in which market value is a function of book value and residual income. Under the non-stationary behavior of these variables, we test the relationship using a cointegration approach to address the
potential spurious relation among them. This approach also avoids the potential problem of specifying the “other information” in the Ohlson model.

Consistent with Qi et al. (2000), we find that market value is not fully cointegrated with book value and residual income. However, by relaxing the constraints in the testing framework from full to fractional cointegration, we are able to find that the relationship exists as specified in the Ohlson model. Therefore, our results lend support to the Ohlson model the way in which market value is related to book value and residual in a slow but mean-reverting process—that is, movements in book value and residual income are slow in “catching-up” with variations in market value.

What might have caused the relationship in the Ohlson model to follow a slow but stationary process? Our additional examination suggests that it is not caused by the violation of the clean surplus relation assumed in the Ohlson model. We find that dirty surplus accounting in 51% of the sample firms that exhibit cointegration of market value with book value and residual income is more severe than that in 49% of the non-cointegrated firms. This result is robust to how we measure dirty surplus items.

We suspect that the answer may come from the information dynamic linkage between current information and future residual income in the Ohlson model. While market value may readily reflect current information, it may take longer for information—especially non-accounting information—to be incorporated into future residual income and hence book value. As a result, the process of explaining market value is slow but stationary.

Appendix

Estimating the relationship between market value, book value, and residual income under fractional cointegration.

Based on Barkoulas et al. (2000), let ARFIMA \((p, d, q)\) denote the model of an autoregressive fractionally integrated moving average process of order \((p,d,q)\), with constant \(\mu\). Using operator notation, it can be expressed as:

\[
\Phi(L)(1 - L)^d(y_t - \mu) = \Theta(L)u_t, \quad u_t \sim i.i.d. (0, \sigma_u^2),
\]

where \(L\) is the backward-shift operator, \(\Phi(L) = 1 - \varphi_1L - \cdots - \varphi_pL^p, \Theta(L) = 1 + \theta_1L + \cdots + \theta_qL^q\), and \((1 - L)^d\) is the fractional differencing operator defined by:

\[
(1 - L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k - d)L^k}{\Gamma(-d)\Gamma(k + 1)},
\]

with \(\Gamma(\cdot)\) denoting the gamma function. The parameter \(d\) takes a real value.

The arbitrary restriction of \(d\) to integer values gives rise to the standard autoregressive integrated moving average (ARIMA) model. If all roots of \(\Phi(L)\) and \(\Theta(L)\) lie outside the unit circle and \(|d| < 0.5\), then the stochastic process \(y_t\) is both stationary and invertible. Assuming that \(d \in (0, 0.5)\) and \(d \neq 0\), Hosking (1981) shows that the correlation function, \(\rho(\cdot)\), of an ARFIMA process is proportional to \(k^{2d-1}\) as \(k \to \infty\).

The autocorrelations of the ARFIMA process consequently decay hyperbolically to zero as \(k \to \infty\), contrary to the geometric decay of a stationary ARMA process. For \(d \in (0, 0.5)\), \(\sum_{j=-n}^{n} |\rho(j)|\) diverges as \(n \to \infty\), and the ARFIMA process is said to exhibit a long memory, or long-range positive dependence. For \(d \in (-0.5, 0)\), it is said that the...
process exhibits intermediate memory, or long-range negative dependence. The process is said to have short memory for \( d = 0 \), corresponding to the stationary and invertible ARMA modeling. For \( d \in (0.5, 1) \), the process is non-stationary (having an infinite variance), but is mean reverting since there is no long-run impact of an innovation on future values of the process.

To estimate the long-memory parameter, we use Robinson’s Gaussian semiparametric method. Robinson (1995) proposes a Gaussian semiparametric estimate, (GS hereafter), of the self-similarity parameter \( H \), which is not defined in closed form. The spectral density of the time series, denoted by \( f(\cdot) \), can be described as:

\[
f(\xi) \sim G_{\xi}^{1-2H} \text{ as } \xi \to 0^+, \tag{12}
\]

for \( G \in (0, \infty) \) and \( H \in (0, 1) \). The self-similarity parameter \( H \) relates to the long-memory parameter \( d \) by \( H = d + 1/2 \). The estimate for \( H \), denoted by \( \hat{H} \), is obtained through the minimization of the function:

\[
R(H) = \ln \hat{G}(H) - (2H - 1) \frac{1}{v} \sum_{z=1}^{v} \ln \xi_z, \tag{13}
\]

with respect to \( H \), where \( \hat{G}(H) = \frac{1}{v} \sum_{z=1}^{v} z^{2H-1} I(\xi_z) I(\xi_z) \) is the periodogram of \( y_t \) at frequency \( \xi_z \), and \( v \) is the number of Fourier frequencies included in estimation (bandwidth parameter). The discrete averaging is carried out over the neighborhood of zero frequency.

Asymptotic theory assumes that \( v \) goes to infinity much slower than \( T \). The GS estimator of \( v^{1/2} \) is consistent. Its variance of the limiting distribution is free of nuisance parameters and equals \( 1/4v \). The GS estimator appears to be the most efficient semiparametric estimator. It remains consistent with the same limiting distribution under conditional heteroscedasticity (Robinson and Henry (1999)).

References

A fractional cointegration approach

Mackinnon JG (2010) Critical values for cointegration tests. Working paper 1227, Department of Economics, Queen’s University